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Algorithms for PMU devices are mostly based on recursive and non-recursive DFT algorithms. Each of these algorithms have advantages and disadvantages in applications of PMU devices. Nowadays, advanced algorithms are based on recursive and non-recursive DFT. Algorithms based on the recursive DFT algorithm are: Fast Fourier Transform (FFT), Interpolated DFT algorithm (IpDFT), DFT Based Demodulation Approach, Clarke Transformation Based DFT and All-Phase Fourier Transform (apFFT). Algorithms based on non-recursive DFT are: Enhanced Phase-Locked Loop (EPLL), Taylor(K)-Kalman Filter, A Novel Modular Positive Sequence Synchrophasor Estimation Algorithm. Third group algorithms which are used for phasor estimation are based on finite impulse response filters (FIR) which are designed by time windows. FIR filters are less commonly used as filters today than filters with infinite impulse response (IIR), so more advanced algorithms based on filters for PMU devices are IIR filters. The method that has the greatest potential for efficient PMU algorithms is wavelet transformation and its variants.

Non-recursive DFT algorithm proved to be more reliable in comparison with recursive DFT algorithm for testing PMU devices. Characteristic of non-recursive DFT is use of new estimation for every new time window. As a result of that, amplitude is constant but phase changes for every new window. Recursive DFT algorithm is based on adding newly calculated phasor to the previously calculated phasor. Algorithms based on FIR filters are designed mostly by Hamming and Blackman window. In comparison with the Hamming window, the Blackman window gives better results. The main reason for that is because errors in frequency and ROCOF are smaller when number of samples is larger than 100. There are also other windows that are not sufficiently investigated such as: Hanning, Nuttall, Blackman-Nuttall, Blackman-Haris, Flat-top, Rice-Vincent etc.

Fast Fourier Transform (FFT) is the most popular advanced algorithm based on a recursive DFT algorithm. FFT has a few advantages: operation on wider range of frequencies and more precise estimation of change in phase angle. Disadvantage is higher cost and complex implementation in DSP processor or other kind of microprocessors. Mathematical expression for FFT is:

$$X[k] = \sum_0^{n-1} x(n)\, e^{-2\pi jnk/N} = \sum_0^{n-1} x(n) W_N^{nk}$$

$$= \sum_0^{(n-1)/2} x(2r) W_N^{k2r} + \sum_0^{(n-1)/2} x(2r+1) W_N^{k(2r+1)}$$

However, the ApFFT algorithm is an advanced algorithm and has better characteristics than FFT. Its good features are based on synchronously estimation of amplitude, phase and frequency. In the spectral domain it also has good properties because ApFFT spectrum use double Hamming window with two spectral lines, and as a result of this, proves excellent data processing and very good elimination of interference at high and semi-high frequencies. The disadvantage is limitation of spectral barrier and not good enough signal to noise ratio (SNR). The IpDFT algorithm has very good characteristics because it estimates amplitude and phase both in static and dynamic conditions at the same time. The minor disadvantage is a little bit higher total vector error (TVE). The DFT based Demodulation Approach is more advanced algorithm based on recursive DFT too. Its main advantage is possibility to reject harmonics in the absence of frequency shift. The disadvantage is very small attenuation at semi-high and high frequencies. The Clarke transformation is used in combination with recursive DFT algorithm as well. Such combination of algorithm has ability to reduce error in wider frequency range. The disadvantage of this method is using Moving Average Filter. Mathematical expressions for Clarke transformations are:

$$X_A(k) = \sqrt{2}A * \cos\left\{\frac{2(f_0 + \Delta f)k\pi}{f_0 N} + \varphi\right\}$$

$$X_B(k) = \sqrt{2}A * \cos\left\{\frac{2(f_0 + \Delta f)\left(k - \frac{N}{3}\right)\pi}{f_0 N} + \varphi\right\}$$

$$X_C(k) = \sqrt{2}A * \cos\left\{\frac{2(f_0 + \Delta f)\left(k + \frac{N}{3}\right)\pi}{f_0 N} + \varphi\right\}$$

Enhanced Phase – Locked Loop is one of the advanced algorithms based on non-recursive DFT. This advanced algorithm solves problems with PLL in a way of constant updating of phasor parameters like amplitude, phase, frequency etc. Consequently, the TVE error is reduced. The disadvantage is a very long processing time.

Taylor^K- Kalman Filter algorithm is the next advanced non-recursive DFT algorithm. The disadvantage is a use of analytical signals. A Novel Modular Positive Sequence Synchrophasor Algorithm has big advantage that is suitable for both types of PMU devices: M and P type. Mode of operation is estimation of the positive synchrophasor phase sequence in several steps: demodulation process, PLL algorithm and Narrow Band Compensation algorithm. The main disadvantage is a very slow data processing.

Algorithms based on filters with infinite impulse response are very advanced algorithms for PMU devices. This type of algorithms are very often constructed as Notch filters. It is very specific for this algorithm that phasor value is obtained in a half-cycle period at the nominal frequency on which it is generated orthogonal component by Notch filter. Distortion (linear and non-linear) is reduced by this algorithm. Disadvantages are slower processing time of estimation phasor parameters because this algorithm uses Goertzel-Zero Crossing Detection (G-ZDC) and operations at half period of frequency which is a result of impossibilities second and other higher harmonics in static conditions. This method can be improved by using Single Amplified Biquad Section (SAB).

Wavelet transformation and its variants provide good estimation of phasor parameters because a lot of signals display values by distributed frequencies while being detected in fixed time interval. Nowadays, recursive wavelet transform is the most frequently used as a type of wavelet transforms for the purpose of estimating phasor parameters. The disadvantage of recursive and other wavelets is complex data processing. Wavelets and its variants have very good possibility for future investigation. The most famous expressions for wavelet function are:

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty$$

$$\psi(\omega)|_{\omega=0} = \int_{-\infty}^{+\infty} \psi(t)dt = 0$$

Nowadays, after implementing and testing many algorithms in PMU devices, ApFFT proved to be the best algorithm for recursive DFT, while combination of Kalman filter and Taylor approximation are the best for non-recursive DFT algorithm. Filters with infinite impulse response have much better characteristics than finite impulse response, so as a result of that they are increasingly used and investigated in phasor estimations. The Wavelet transform and its variants provide the best opportunity for further research in the field of algorithms for estimating the value of synchrophasors.